OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **2644**

Probability & Statistics 4

Wednesday **22 JUNE 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1 The random variable X has mean $\frac{1}{8}$ and its moment generating function, $\mathrm{M}(t)$, is $a\left(\mathrm{e}^{2bt} + \mathrm{e}^{-bt}\right)$, where a and b are constants.

(i) Find the value of a. [1]

(ii) Show that $b = \frac{1}{4}$. [3]

2 The events A, B, C are such that A and B are independent, B and C are independent, and C and A are independent. It is given that $\mathrm{P}(A) = \mathrm{P}(B) = \mathrm{P}(C) = \frac{1}{2}$ and $\mathrm{P}(A \cup B \cup C) = \frac{15}{16}$.

(i) Write down the formula for $\mathrm{P}(A \cup B \cup C)$ in terms of the probabilities of the events A, B, C, $A \cap B$, $B \cap C$, $C \cap A$, and $A \cap B \cap C$. [1]

(ii) Hence find $\mathrm{P}(A \cap B \cap C)$. [3]

(iii) Show that the events C and $A \cap B$ are not independent. [2]

3 Metal used in the bodies of aircraft contains titanium as a strengthening agent. The specification is that the average titanium content should be 8.5%. A batch of the metal was received by the manufacturer and 13 randomly selected samples were analysed. The percentages of titanium in the samples were as follows.

8.40 8.63 8.17 8.72 8.42 8.61 8.65 8.70 8.38 8.59 8.53 8.68 8.75

(i) State a distributional assumption needed for a single-sample Wilcoxon test to be valid. [1]

(ii) Using this Wilcoxon test (which may be assumed to be valid), test at the 5% significance level whether the specification is being met. [7]

4 The (positive) random variable S is such that S^2 is an unbiased estimator of a (non-zero) population variance σ^2.

(i) Express this statement in terms of an expectation. [1]

(ii) Express $\mathrm{Var}(S)$ in terms of $\mathrm{E}(S)$ and $\mathrm{E}(S^2)$. [1]

(iii) Hence show that S cannot be an unbiased estimator of σ, determining whether, on average, S will overestimate or underestimate the value of σ. [5]

5 A fair cubical die has one face numbered 1, two faces numbered 2 and three faces numbered 3. The die is thrown three times. The largest and smallest values obtained are the random variables L and S respectively. The joint probability distribution is given in the following table.

		L		
		1	2	3
	1	$\frac{1}{216}$	$\frac{1}{12}$	$\frac{1}{3}$
S	2	0	$\frac{1}{27}$	$\frac{5}{12}$
	3	0	0	$\frac{1}{8}$

(i) Show how the value $\frac{5}{12}$ in the table is obtained. [2]

(ii) Find $E(L)$ and $E(S)$. [3]

(iii) Find $E(LS)$. [2]

(iv) Find $Cov(L, S)$. [2]

6 The independent random variables X and Y have Poisson distributions with means λ and μ respectively.

(i) Show that the probability generating function of X is $e^{-\lambda(1-t)}$. [3]

(ii) Hence show that the sum of a random observation of X and a random observation of Y has a Poisson distribution. [2]

(iii) Give a reason why the difference of these random observations does not have a Poisson distribution. [1]

The number of e-mails received by Ms Jones during a randomly chosen morning has a Poisson distribution with mean 2.8. During a randomly chosen afternoon the number of e-mails she receives has an independent Poisson distribution with mean 3.2.

(iv) Find the probability that Ms Jones receives a total of 5 e-mails during a randomly chosen day. [3]

(v) Given that Ms Jones received 5 e-mails during a randomly chosen day, find the probability that she received more e-mails in the morning than she did in the afternoon. [4]

[Question 7 is printed overleaf.]

7 Two populations, A and B, have medians m_A and m_B respectively. The Wilcoxon rank sum test based on independent random samples is used to test for a possible difference between m_A and m_B.

(i) State any assumption about the distributions of A and B needed for the validity of the test. [1]

A manufacturer of potato crisps has developed a new crisp which is intended to be less noisy when eaten than the current variety. A study was carried out using 16 volunteers who were divided randomly into two equal groups. The members of one group were given a handful of the current variety (A) and the members of the other group were given a handful of the new variety (B). They were told to chew the crisps with their mouths closed. The loudness of their chewing was measured and the peak values, in decibels to the nearest db, were as follows. The values are ordered for convenience.

Crisp A	36	43	47	48	51	60	63	74
Crisp B	22	26	34	42	50	58	59	61

(ii) Use a Wilcoxon test, at the 5% significance level, to test whether the new crisps are less noisy than the current crisps. [7]

(iii) Using a normal approximation to the test statistic W, find $P(W \leqslant w)$, where w is the value obtained in part **(ii)**. [5]

1	(i)	M(0) = 1; $a = 0.5$	B1	1	
	(ii)	$M'(t) = a(2be^{2bt}-be^{-bt})$	M1		aef
		$t = 0$, $ab = 0.125$	M1		
		$b = 0.25$	A1	3	Only if a found
2	(i)	$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$ $- P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$	B1	1	
	(ii)	Substitute ¼ for $P(A \cap B)$ etc and other probs	M1A1		
		Answer $^3/_{16}$	A1	3	
	(iii)	Show EITHER $P(A \cap B \mid C) \neq P(A \cap B)$			
		OR: $P(A \cap B \cap C) \neq P(A \cap B)P(C)$			
		OR: $P(A)P(B)P(C) \neq P(A \cap B)P(C)$	M1A1ft	2	ft $^3/_{16}$
3	(i)	Distribution of % titanium symmetric	B1	1	
	(ii)	H_0: $m = 8.5$, H_1: $m \neq 8.5$	B1		Not mean or average
		Subtract 8.5 and rank	M1		Or subtract from 8.5
		-0.10 0.13 -0.33 0.22 -0.08 0.11 0.15 0.20 -0.12 0.09 0.03 0.18 0.25 4 7 13 11 2 5 8 10 6 3 1 9 12			A1
		$Q = 25$ ($P = 66$)	B1		
		Use 25	B1ft		ft P,Q
		5% critical region: $T \leq 17$	B1		
		Insufficient evidence that average is not .5%	B1ft	7	ft 25
		SR: 1-tail test: B0M1A1B1B1B1B1			critical value 21
4	(i)	$E(S^2) = \sigma^2$	B1	1	
	(ii)	$Var(S) = E(S^2) - [E(S)]^2$	B1	1	Or $E(S^2) - \mu^2$
	(iii)	EITHER $Var(S) > 0$; $[E(S)]^2 < \sigma^2$	M1M1		
		$\Rightarrow E(S) < \sigma$	A1		
		S is not an unbiased estimator of σ	A1		
		Underestimates σ on average	A1	5	
		OR: S unbiased estimator $\Rightarrow E(S) = \sigma$	M1		
		$\Rightarrow Var(S) = 0$ which is untrue			
		$\Rightarrow S$ is not an unbiased estimate of σ	A1		
		$Var(S) > 0 \Rightarrow [E(S)]^2 < \sigma^2$	M1		
		$\Rightarrow E(S) < \sigma$	A1		
		$\Rightarrow S$ underestimates σ	A1	5	B1/3 for unsupported statement
5	(i)	Consider 2,2,3 and 2,3,3 with permutations	M1		
		3(⅓)(⅓)(½) + 3(½)(½)(⅓)	A1	2	
	(ii)	Correct method for finding $E(L)$ or $E(S)$	M1		
		$E(L) = {}^{155}/_{54}$; $E(S) = {}^{46}/_{27}$	A1A1	3	

	(iii)	$1(^1/_{216})+2(^1/_{12})+3(^1/_3)+4(^1/_{27})+6(^5/_{12})+9(^1/_8)$ $= {}^{89}/_{18}$	M1 A1	2	Allow one error
	(iv)	${}^{89}/_{18} - ({}^{46}/_{27})({}^{155}/_{54})$	M1		Not E(*LS*)=E(*L*)E(*S*)
		$= {}^{79}/_{1458}$	A1	2	art 0.0542
6	(i)	$\sum e^{-\lambda}\lambda^x t^x/x!$ for x=0 to ∞	M1		Allow finite, or no x=0
		$=e^{-\lambda}\sum(\lambda t)^x/x!$	A1		
		$=e^{-\lambda}e^{\lambda t} = e^{-\lambda(1-t)}$	AG A1	3	
	(ii)	$e^{-\lambda(1-t)}\ e^{-\mu(1-t)}$	M1		
		$=e^{-(\lambda+\mu)(1-t)}$ => Po($\lambda+\mu$)	A1	2	Correct form
	(iii)	EITHER: $X - Y$ takes negative values			
		OR: $E(X-Y) = \lambda - \mu$, $Var(X-Y) = \lambda + \mu$, unequal	B1	1	
		OR: $E(X-Y)$ could be negative			
	(iv)	Use Po(6) to find $P(X + Y = 5)$	M1		λ=6
		EITHER $e^{-6}6^5/5!$ OR 0.4457 – 0.2851	M1		Attempt at 6
		0.1606	A1	3	art 0.161
	(v)	$P(X=3,Y=2$ or $X=4,Y=1$ or $X=5,Y=0)$	M1		Allow one pair missing
		$e^{-6}[(2.8^3/3!)(3.2^2/2!)+(2.8^4/4!)3.2+2.8^5/5!]$	A1		
		$\div e^{-6}6^5/5!$	M1		
		$0.4377 \approx 0.438$	A1	4	Cao
7	(i)				
		A and *B* have identical (continuous) distns (apart from location).	B1	1	
	(ii)	H_0: $m_A = m_B$, H_1: $m_A > m_B$	B1		Allow medians
		Rank samples	M1		
		22 26 34 36 42 43 47 48 50 51 58 59 60 61 63 74 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 *B B B A B A A A B A B B A B A A*			A1
		$R_A = 79, R_B = 57$	B1		
		$W = 57$	B1ft		ft R_A, R_B
		5% critical region, $W \le 51$	M1		
		Accept new variety not less noisy than old	A1ft	7	ft W
	(iii)	$W \sim N(68, 90⅔)$	B1B1		Mean and variance
		$\pm[57.5 - 68]/\sqrt{(90⅔)}$	M1		With or without cc
		–1.103	A1ft		ft μ, σ with cc
		0.135	A1	5	